The Science of Grilling Meat

With summer coming on more of us will be spending time with the grill. Certainly the first concern when grilling meat is not to char the outside of the meat, but the second is to bring the inside to the desired temperature.

The following table tells us the temperature we are shooting for, given the preferences of ourselves or our guests. Our problem is to find the time it will take for the center of the steak to reach this temperature, starting say from refrigerator temperature, and having a grill that is at a known temperature. Table 1 shows the final temperatures commonly desired when grilling steak and ground beef.

The temperature distribution and heat flow in a steak or ground beef patty cooking on a grill is pretty complex. It could be modeled on a computer, but that's not what we want here.

We can approximate it with a solution to a highly idealized steak cooking situation.

Assume you have a thick slab of meat initially at temperature T_o. At time 0 one surface of the meat is placed in contact with the grill surface at temperature T_g. For this simple condition the temperature of the meat for any time, *t* and any depth, *z* beneath the heated surface can be computed from

$$\frac{T(z,t) - T_o}{T_g - T_o} = erfc\left(\frac{z}{2\sqrt{Dt}}\right) \qquad (1)$$

Here *D* is the thermal diffusivity of the meat and *erfc* is the complementary error function which can be found in tables or approximated numerically (Press et al. 1989).

Table 1. Beef and Lamb Cooking Temperatures

Roasts, Steaks	Temperature	Description
Rare	120° to 125°F	Center is bright red, pinkish toward the exterior portion
Medium Rare	130° to 135°F	Center is very pink, slightly brown toward the exterior portion
Medium	140° to 145°F	Center is light pink, outer portion is brown
Medium Well	150° to 155°F	Not pink
Well Done	160°F and above	Steak is uniformly brown throughout
Ground Meat	160° to 165°F	No longer pink but uniformly brown throughout

We want to know the time it will take to bring the center of the steak to the desired temperature. We can therefore take z to be the half thickness of the steak, $T(z,t)$ as the temperature from Table 1, T_o as the initial temperature of the steak, and T_g as the grill temperature.

The diffusivity must be measured. Thermal diffusivity is the ratio of thermal conductivity to volumetric specific heat of a substance. The KD2-Pro thermal properties analyzer measures both thermal conductivity and volumetric specific heat using two needles, one with a heater and the other with a temperature sensor. A heat pulse applied to the needle is sensed by the sensor in the adjacent needle. By analyzing the temperature response the thermal properties of the material can be determined.

Equation (1) can be solved to determine the time required for the meat to reach the desired temperature:

$$t = \frac{z^2}{4D\left[erfc^{-1}\left(\frac{T(z,t)-T_o}{T_g-T_o}\right)\right]^2} \qquad (2)$$

The only hard part of this equation is finding values for the inverse complimentary error function of the temperature ratio in parentheses in equation 2. Values are given in Table 2 for the range of temperatures likely to be encountered in cooking steaks on a grill.

Table 2. Inverse complementary error function for values of the temperature ratio likely to be encountered in grilling meat. The ratio x is the left hand side of equation 1 and in parentheses in equation 2.

x	$erfc^{-1}(x)$
0.1	1.16
0.2	0.91
0.3	0.73
0.4	0.59
0.5	0.48
0.6	0.37
0.7	0.27
0.8	0.17
0.9	0.09
1	0.05

As an example, assume we have meat starting at 35 F. We want to serve it medium rare, which, from Table 1 is 130 F. Assume the grill temperature is 375 F.

$$\frac{T(z,t)-T_o}{T_g-T_o} = \frac{130-35}{375-35} = 0.29 = x$$

From Table 2 the corresponding inverse complementary error function value is about 0.75. Its squared value is 0.56. The diffusivity is easiest to deal with in metric units, so the meat thickness should also be in metric. Assume a steak about 1" thick. Its half thickness is therefore around 12 mm. The numerator in eq. 2 is 144 mm^2. If we measure the diffusivity of the meat and find it to be 0.15 mm^2/s, then the cooking time is

$$t = \frac{144\,mm^2}{0.56 \times 0.15 mm^2/s} = 1706\,s$$

or about 28 minutes.